

## **DISTURBANCE REJECTION CONTROL APPLIED TO A GANTRY CRANE**

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### **ABSTRACT**

*The paper highlights a novel method for controlling a gantry crane model based on an Active Force Control (AFC) strategy. It is a disturbance rejection control technique in which AFC is employed to control accurately and robustly the trolley part of the crane along a desired path, compensating at the same time the payload sway at the end of the traversed motion. AFC is designed and implemented using a two degree-of-freedom controller-the outer classic Proportional-Integral-Derivative (PID) control loop provides the commanded signal while the internal AFC loop accommodates the known and unknown disturbances present in the gantry system. Results from the simulation study clearly show that the crane can perform its predefined task faster with a minimum payload sway angle compared to the PID control method.*

**Keywords:** *Active force control, disturbance rejection, PID control, gantry crane*

### **1.0 INTRODUCTION**

In harbours, large factories, building and construction sites, it is common to see gantry cranes, which are primarily being used to grip, lift and transport heavy payloads (usually containers) to the desired locations as fast and as accurate as possible. This is done using the cabled hoisting mechanism typically equipped into the crane through a combination of a hoisting line (cable) and a hook. The payload is usually attached to the hook and suspended from a point on the support mechanism (trolley) that moves the suspension point around the gantry crane workspace, while the hoisting mechanism lifts and lowers the payload to avoid any obstacles in its path and finally the payload is deposited at the target location. Figure 1 shows a snapshot of a gantry crane system that can be typically found in a factory.

Recently designed gantry cranes are larger and have higher lifting capacities and travel speeds. To achieve high productivity and to comply with the safety requirements, these cranes require effective controllers such as anti sway controls. Most of the anti sway control of the gantry crane is performed manually by skillful

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human operators who combine their intuition, experience and skill to manipulate a load hanging on a hoisting cable by stopping the trolley near the desired position and then letting the payload to stop oscillating gradually by a further gentle movement of the trolley. However, this poses some practical problems, particularly due to human fatigue that in turn may affect significantly the performance and operation of the gantry crane. In addition, gantry cranes are mostly equipped with cabled hoisting mechanism, which are prone to the load sway problems due to the fact that the assembly of the cable-hook-payload results in complex system dynamics. Even in the absence of external disturbances, inertia forces due to the motion of the crane can induce significant payload oscillations. This problem is exacerbated by the fact that gantry cranes are usually lightly damped, which implies that any transient motion takes a long time to dampen out [2]. For this reason, the payload sway angle should be kept to a minimum; otherwise, a large payload sway angle during transportation may cause damage to the payload, surrounding equipment or even personnel. If the pendulum-like oscillations of the payload can be constrained using an appropriate method, there will be a number of benefits such as having greater yield and safety margin, enabling higher operating speed, enhancing work quality and creating greater throughput for a given installed capacity. Besides, most actual systems are influenced by noise and external disturbances including gantry crane. These disturbances such as wind, unstable mounting and others may degrade the performance of the gantry crane.

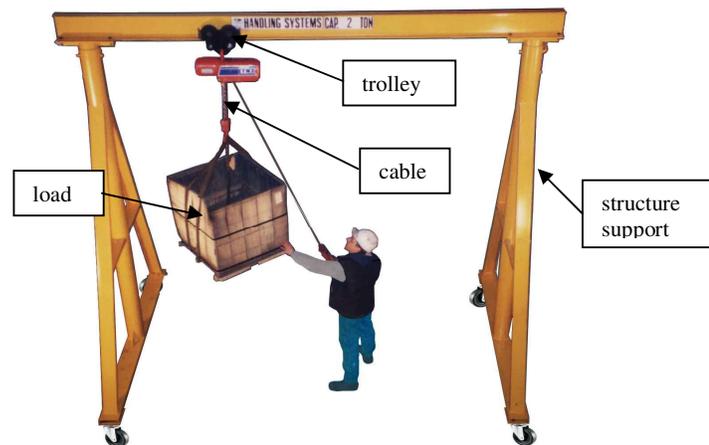


Figure 1: A gantry crane system [1]

Previously, many approaches have been proposed to solve the above stated problem. An approach known as input shaping technique is widely used as an open-loop control strategy for gantry cranes. The controller using this strategy will accelerate the trolley in steps of constant acceleration. Although there will be effectively no residual oscillations but large transient oscillations will happen during the transportation. Besides, input-shaping techniques are limited by the

facts that (i) they are sensitive to variations in the parameter values about the nominal values and changes in the initial conditions and external disturbances and (ii) they require 'highly accurate values of the system parameters' to achieve satisfactory system response [3].

Another approach is known as wave-based technique in which the control mechanism, in the form of mechanical waves, is conceptualised. If the motion of the trolley is seen as launching mechanical waves into, or absorbing waves out of the system, there is a greater hope of achieving the position control and vibration absorption successfully. Here 'waves' are taken not so much in the usual understanding as periodic motions in space and time, but more as propagating, long-wavelength disturbances that may or may not be oscillatory in nature. Such wave-based models are closer to the physics of the problem and allow characteristic features of the system dynamics to be used positively and creatively [4].

Feedback or closed-loop control methods have also been introduced for the control of gantry crane. One of them is the PID controller used for both position and anti-sway controls. However, it usually invokes higher steady state error and low sensitivity to disturbance. In addition, due to the existence of friction, gravity forces and other uncertainties, the PID control cannot guarantee that the steady state error becomes zero [5]. On top of that, PID controller needs to be designed based on the model and parameters of the plant. The modeling and parameter identification processes are tedious [6] yet the performance is susceptible to external disturbances. Most of the control methods, especially classical controls, will be able to provide great performance only under low speed operation and without any disturbance acting on the system. Although closed-loop control may be used to alleviate some of the aforesaid problems, it typically requires a very accurate plant model and cannot therefore offer significant improvements over the open-loop control [7]. As the operational speed increases or in the event of any disturbance, the performance of such controller considerably degrades and deteriorates. It is usual that at times, the actual system is forced to operate at high speeds and in the presence of various forms of loading conditions (disturbances). Thus, a robust and effective method with the ability to alleviate the previously discussed problems should be designed and implemented. Here, a novel control method using active force control (AFC) strategy is proposed. AFC is very practical, not computationally intensive, and able to produce superior performance if a set of simple criteria and conditions is fulfilled as demonstrated in a number of studies [8-10]. It is practically and readily implemented in real-time due to the simplicity in its control algorithm and it operates on the physical measurements or approximations of the relevant parameters. Further consolidation of the AFC technique through the use of artificial intelligence (AI) and adaptive elements applied to a number of dynamical systems has been extensively reported [11-13]. Thus, the main aim of this paper is to demonstrate the effectiveness and novelty of the AFC scheme to control a highly non-linear gantry crane system.

## 2.0 SYSTEM MODELLING OF GANTRY CRANE

A schematic diagram of a gantry crane model is shown in Figure 2. Generally the configuration of this model is specified by the horizontal position of trolley,  $x$ , the length of the hoisting cable,  $l$ , and the sway angle, theta or  $\theta$ . A number of assumptions are made in order to simplify the system:

- The trolley and the payload can either move or oscillate in x-y plane.
- The tension force that will cause the hoisting cable to elongate is neglected.
- Both the trolley and the payload are considered as point masses.
- The friction between the trolley and the rail is neglected.

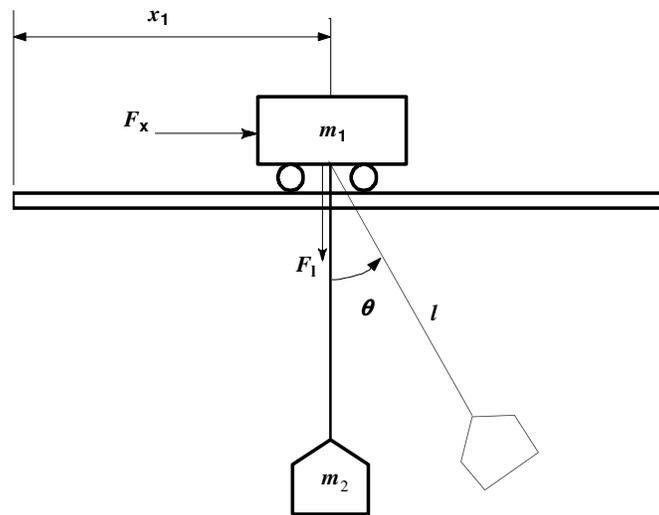


Figure 2: Schematic diagram of a gantry crane model

By using the *Lagrange's* equation, the equations of motion for the gantry crane model associated with the generalized coordinates can be summarized as:

$$F_x = (m_1 + m_2) \ddot{x} + m_2 l \ddot{\theta} \cos \theta + m_2 \dot{l} \dot{\theta} \sin \theta + (2m_2 \dot{l} \cos \theta - m_2 l \dot{\theta} \sin \theta) \dot{\theta} \quad (1)$$

$$l \ddot{\theta} + 2 \dot{l} \dot{\theta} + \ddot{x} \cos \theta + g \sin \theta = 0 \quad (2)$$

$$F_l = m_2 \ddot{l} + m_2 \ddot{x} \sin \theta - m_2 l \dot{\theta}^2 - m_2 g \cos \theta \quad (3)$$

The above nonlinear equations can be represented in the following matrix form:

$$\begin{pmatrix} F_x \\ 0 \\ F_l \end{pmatrix} = \begin{bmatrix} m_1 + m_2 & m_2 l \cos \theta & m_2 \sin \theta \\ \cos \theta & l & 0 \\ m_2 \sin \theta & 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \\ \ddot{l} \end{bmatrix} + \begin{bmatrix} 0 & m_2 \dot{l} \cos \theta - m_2 l \dot{\theta} \sin \theta & m_2 \dot{\theta} \cos \theta \\ 0 & \dot{l} & \dot{\theta} \\ 0 & -m_2 \dot{l} & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{l} \end{bmatrix} + \begin{bmatrix} 0 \\ g \sin \theta \\ -mg \cos \theta \end{bmatrix} \quad (4)$$

The equations of motion are nonlinear because of the presence of trigonometric terms such as  $\sin \theta$  and  $\cos \theta$  as well as the quadratic term contained in the equations. To perform linearization, the *Taylor* series expansion is used. Assuming that the sway angle is small, we have:

$$\theta_0 = 0 \quad \text{and} \quad \sin \theta \approx \theta$$

As for the trigonometric  $\cos \theta$ , it can be expressed as:

$$\cos \theta \approx \cos \theta_0 - \sin \theta_0 (\theta - \theta_0)$$

Since the sway angle is small, we have:

$$\theta_0 = 0 \quad \text{and} \quad \cos \theta \approx 1$$

Besides, since  $\dot{\theta}$  is also small, thus the quadratic term  $\dot{\theta}^2$  is correspondingly small and thus can be neglected, i.e.:

$$\dot{\theta}^2 \approx 0$$

The tension force of hoisting cable which cause the cable to elongate is neglected, thus the length of the cable can be assumed to be constant such that:

$$\dot{l} = \ddot{l} = 0$$

The equations of motion for a linearized model of a gantry crane are represented as follows:

$$F_x = (m_1 + m_2) \ddot{x} + m_2 l \ddot{\theta} \quad (5)$$

$$l \ddot{\theta} + \ddot{x} + g \theta = 0 \quad (6)$$

$$F_l = m_2 \ddot{x}\theta - m_2 g \quad (7)$$

### 2.1 Representation of Models in Laplace or s Domain

In order to represent Equations (5) to (7) in  $s$  domain, their respective transfer functions can be derived through *Laplace* transformation assuming the initial states to be zero. Thus for Equation (5), it becomes:

$$F_x(s) = (m_1 + m_2)s^2 X(s) + m_2 l s^2 \theta(s) \quad (8)$$

Equation (8) then becomes:

$$l s^2 \theta(s) + s^2 X(s) + g \theta(s) = 0 \quad (9)$$

Rearranging Equation (9), we get:

$$\theta(s) = \frac{-s^2 X(s)}{l s^2 + g} \quad (10)$$

And

$$X(s) = \left[ \frac{-l s^2 \theta(s) - g \theta(s)}{s^2} \right] \quad (11)$$

Substituting Equation (10) into Equation (8), a transfer function is obtained as follows:

$$\frac{X(s)}{F_x(s)} = \frac{l s^2 + g}{m_1 l s^4 + s^2 g (m_1 + m_2)} \quad (12)$$

Similarly, substituting Equation (11) into Equation (8), another transfer function is derived as follows:

$$\frac{\theta(s)}{F_x(s)} = \frac{1}{-m_1 l s^2 - (m_1 + m_2)g} \quad (13)$$

### 2.2 Derivation of Actuator Transfer Function

A common actuator used in many control systems is the DC motor. It can provide rotary motion and if it is coupled to wheels or drums and hoisting cables, a translational motion can be produced. By applying *Kirchoff's* voltage law, we obtain the following equation:

$$V(t) = I(t)R + L \frac{dI(t)}{dt} + K_m \dot{\theta}(t) \quad (14)$$

By applying *Laplace* transformation to Equation (14):

$$V(s) = I(s)R + LsI(s) + K_m s\theta(s) \quad (15)$$

Also, the torque produced by the motor is the product of torque constant and current. This can be expressed as:

$$T(t) = K_t I(t) = J \ddot{\theta}(t) + b \dot{\theta}(t) \quad (16)$$

By using *Laplace* transformation, Equation (16) becomes:

$$I(s) = \frac{Js^2\theta(s) + bs\theta(s)}{K_t} \quad (17)$$

Substituting Equation (17) into Equation (15), the resulting transfer function is as follows:

$$\frac{\theta(s)}{V(s)} = \frac{K_t}{LJs^3 + s^2(Lb + RJ) + s(K_t K_m + Rb)} \quad (18)$$

### 2.3 Derivation of the Drive Mechanism Transfer Function

In order to transfer the angular motion of the motor to translational motion, a rack and pinion mechanism is used. The governing equation is:

$$T(t) = J \ddot{\theta}(t) + b \dot{\theta}(t) = F_E(t)r \quad (19)$$

By using *Laplace* transformation, the following equation is obtained:

$$\frac{F_E(s)}{\theta(s)} = \frac{Js^2 + bs}{r} \quad (20)$$

Using Equations (12), (13), (18), (20) and those from the controller models (PID and AFC), the system is simulated extensively using MATLAB/Simulink platform.

### 3.0 ACTIVE FORCE CONTROL (AFC)

Active Force Control is based on the well-known principle of invariance as described in [14]. A rigorous mathematical formulation including the stability analysis of the AFC scheme can be found in [8, 14]. It is an effective way to eliminate the external disturbance practically. It is practical because the mathematical complexity is reduced significantly since it operates either on the physical measurements of relevant parameters or on the estimated parameters.

Besides, the computational burden is much reduced and hence it can be easily implemented in real-time. The underlying principle of AFC is that it relies heavily on the estimated and measured parameters. For a rotary system, there is a need to measure the torque (using a torque sensor) and the angular acceleration (using an accelerometer) in order to indirectly estimate the disturbance torque. Likewise, for a translation system, it is required to measure the force using a force sensor and the acceleration using an accelerometer to estimate the disturbance force required in the AFC loop. The acquisition of the estimated disturbance force or torque will subsequently result in the compensation of the actual disturbances, thereby assuring the excellent performance of the AFC method. For the gantry crane system used in the study with reference to the trolley component and from *Newton's* second law of motion, we have:

$$\Sigma F = ma \tag{21}$$

Based on Equation (21), a dynamic model of the trolley component can be derived as described in the previous section of this paper. An AFC schematic is shown in Figure 3. Note that a generic controller is usually used in the AFC scheme. A common one is the classic PID controller typically represented by the following equation:

$$G_c(s) = K_P + K_I/s + K_D s \tag{22}$$

The commanded trajectory in this study is the desired position of the trolley and sway angle of the payload. The method used to estimate the estimated mass is through crude approximation method based on the dynamic model of the crane as mentioned in [9].

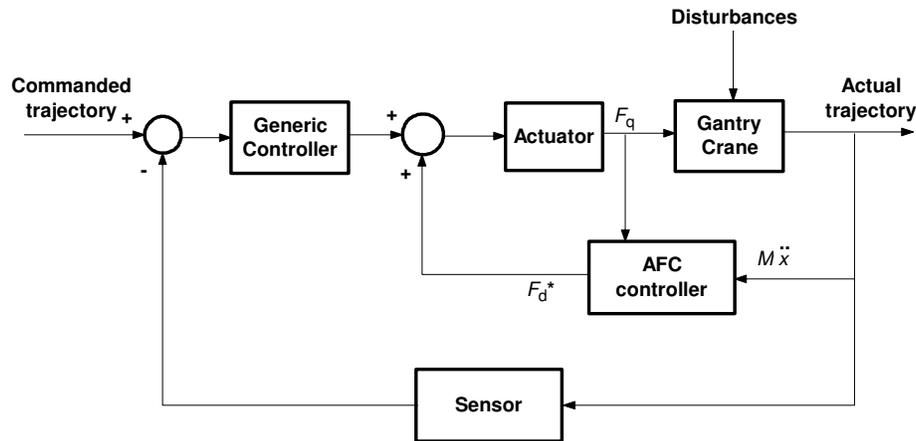


Figure 3: AFC scheme applied to control a gantry crane

The most important equation for the AFC scheme is to obtain the estimated disturbance force in the AFC loop as follows:

$$F_d^* = F_q - M \ddot{x} \quad (23)$$

If Equation (23) can be ascertained through appropriate measured or estimated means, a very robust and stable performance is often assured. Various forms of disturbances or loading conditions are considered to test the system performance and robustness. Note that in the simulation study the ‘measured’ parameters are assumed to be perfectly modelled, i.e. direct values from the simulation are used by assuming the transfer functions of the ‘sensors’ to be unity.

#### 4.0 RESULTS AND DISCUSSION

In this study, the simulation work is performed by using the MATLAB and Simulink software packages. The simulation block diagrams for the AFC schemes consist of a number of components and subsystems including the PID controller, AFC loop, gantry crane dynamic model and the disturbance model. The disturbance model is comprised of several external disturbances such as the constant external force, harmonic force and impulse force to test the robustness of the system. The input to the system, i.e. desired position is in the form of a step function considering 1 m length of trolley linear displacement along the rail. A Simulink block diagram of the gantry crane with the proposed control system is produced as shown in Figure 4. Also, an animation of the proposed system using MATLAB with suitable graphic user interface (GUI) has been designed and developed to clearly observe the system responses as shown in Figure 5.

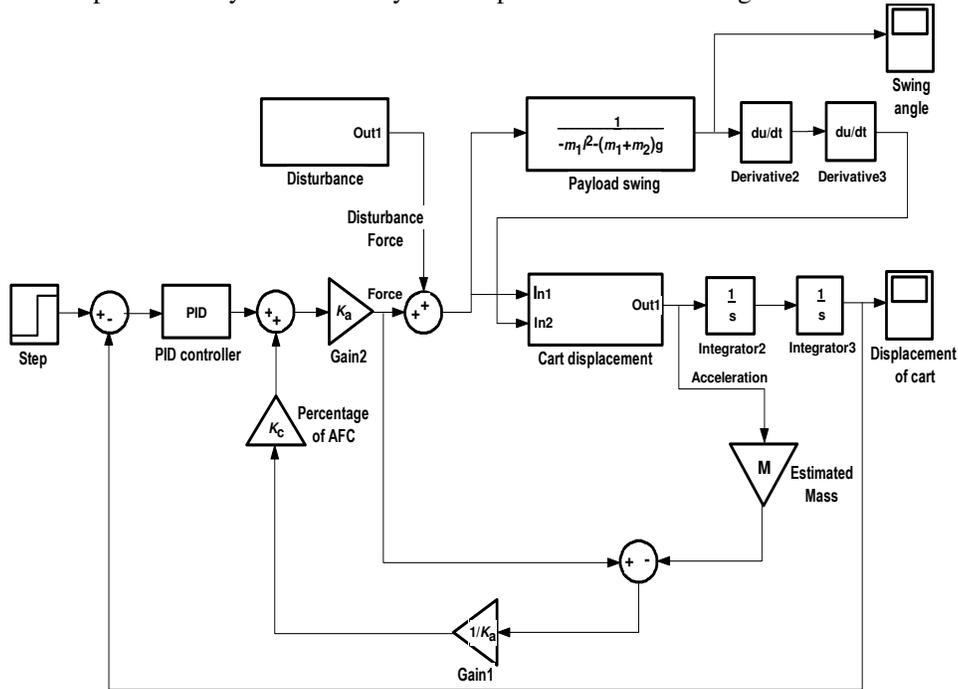


Figure 4: Simulink diagram of the proposed system

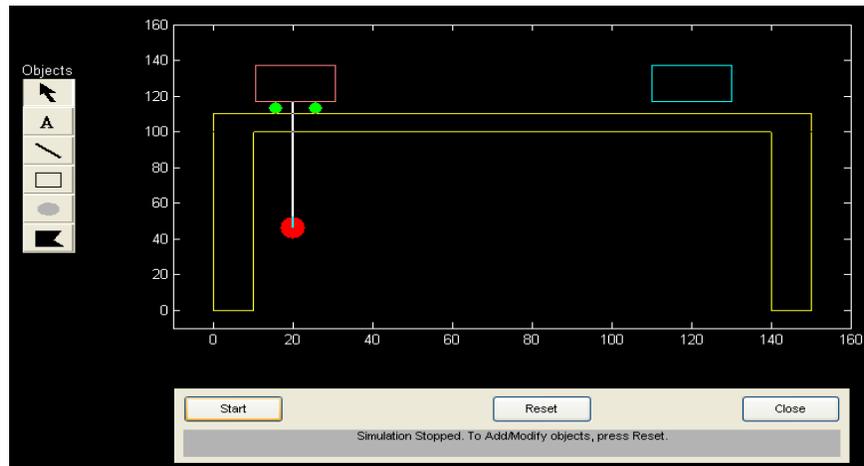


Figure 5: Animation program window

The parameters utilized in the simulation are as follows:

*Gantry crane parameters:*

Mass of trolley = 0.536kg  
 Mass of payload = 0.375kg  
 Cable length = 0.64m

*Controller parameters:*

For PID control:

Controller gains:  $K_P=5$ ,  $K_I=1$ ,  $K_D=4$

For AFC control:

Controller gains:  $K_P=3$ ,  $K_I=0$ ,  $K_D=7.9375$

Motor torque constant,  $K_t= 0.00767\text{N/A}$

Note that all the above gains are assumed to be appropriately tuned using a suitable heuristic method.

*Simulation parameters:*

Integration algorithm: ODE45 (Dormand–Prince)

Simulation time start: 0.00 s

Simulation time stop: 20.00 s

In order to further examine the effectiveness and robustness of the AFC schemes, a number of external disturbances are introduced in the simulation. The disturbances considered in the study are:

- Constant force = 5 N
- Harmonic force =  $5 \sin 10t$  N
- Impulse force = 5 N, 10% duty cycle of period 2 s

The graphical results related to the displacements of trolley ( $x$ ) and sway angles ( $\theta$ ) of the payload under different control schemes and loading conditions

(disturbances) are shown in Figures 6 to 13. Note that for all figures, the responses of the PID controller are characterized by the dotted lines while those of AFC schemes represented by solid lines.

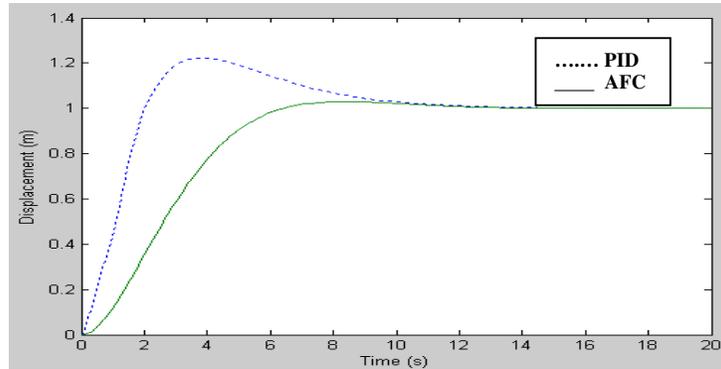


Figure 6: Trolley displacement, no disturbance: PID and AFC

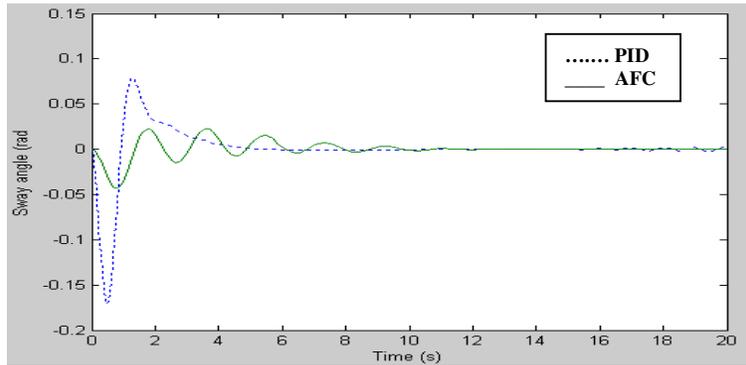


Figure 7: Payload sway angle, no disturbance: PID and AFC

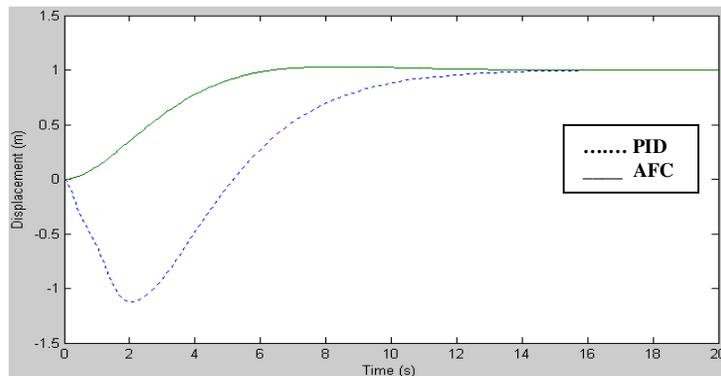


Figure 8: Trolley displacement, constant force

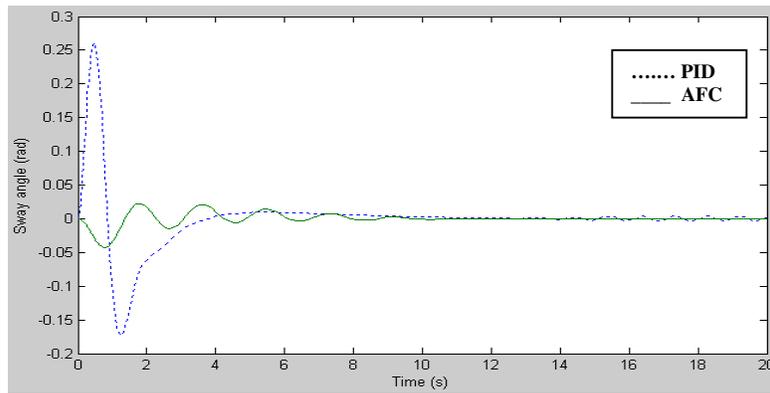


Figure 9: Payload sway angle, constant force

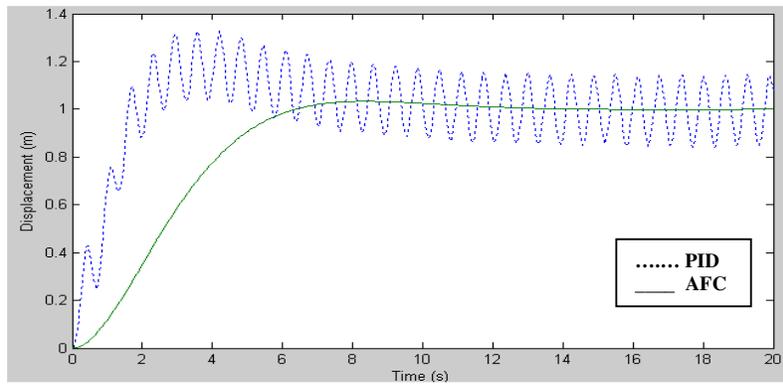


Figure 10: Trolley displacement, harmonic force

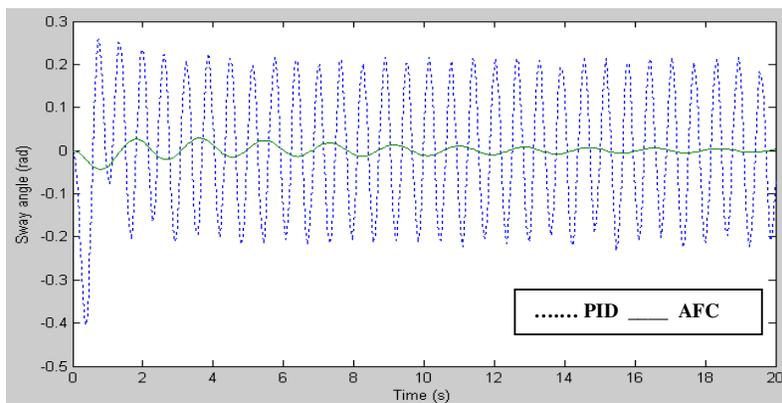


Figure 11: Payload sway angle, harmonic force

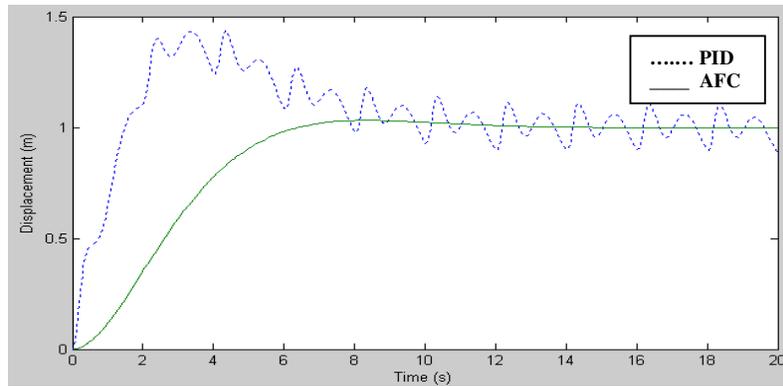


Figure 12: Trolley displacement, impulse force

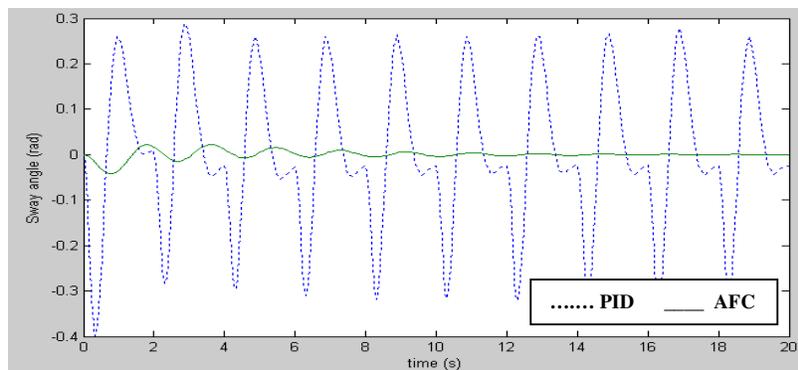


Figure 13: Payload sway angle, impulse force

The performance of the AFC scheme can be evaluated and compared (against the classic PID controller) through the responses obtained from the trolley displacements and payload sway angles. For the PID controller, it is observed that it is able to perform quite well if there is no disturbance force acting on the gantry crane. In fact, the rise time for the trolley movement is found to be faster (albeit with a slight overshoot) than the AFC scheme as depicted in Figure 6. Also, there is very little oscillation of the sway angle although large initial swaying movement is observed as shown in Figure 7. However, at the onset of applying any type of disturbances, the performance of the PID controller drops drastically. Figure 8 shows the degradation of the controller performance in controlling the displacement of the trolley subject to a constant force and correspondingly this results in a larger initial sway angle (Figure 9). In contrast, the AFC scheme is relatively unchanged and unperturbed even with the presence of the disturbances. Generally, for the PID controller, the control of the displacement of the trolley is relatively better (apart from Figure 10) than the sway angle which is characterized by persistent oscillation or ‘hunting trend’ (Figures 11 and 13) due to the nature of the introduced disturbances. For example, the harmonic force, which is sinusoidal in nature, results in the continuous oscillation pattern of the response. As expected,

the performance of the PID controller is very predictable since the controller is widely known to be not robust and quite vulnerable in adverse operating conditions. This is also due to the fact that PID controller gains are normally designed to be fixed for all conditions and thus are not optimized or adapted for different settings, environments, and conditions. On the other hand, it is easily seen that the overall performance of the AFC schemes with or without disturbances outperforms that of the PID counterpart in most cases. Considering similar operating environments, the steady state errors for all the cases almost approach the zero datum with a little fluctuation and hardly no overshooting. Thus, it can be deduced that the proposed AFC scheme demonstrates a high degree of accuracy and robustness even under the influence of various disturbances. The error level produced by the AFC scheme is way below the 5% margin compared to the PID controller. The resulting sway angles produce little oscillations compared to those of the PID controller as illustrated in Figures 11 and 13. The results can be further corroborated by observing the visible responses through the developed animation program.

## **5.0 CONCLUSIONS**

A simple and novel method based on disturbance rejection control technique to control a gantry crane system has been demonstrated. Under various loading and operation settings, simulation results have demonstrated the superiority of performance of the gantry crane system with AFC-based scheme compared to that of the PID counterpart. This clearly implies that the AFC method is very effective and robust even in the presence of the introduced disturbances. On top of that, the AFC strategy does not require excessive computational load due to the implementation of simple control algorithms. However, the procedure to tune the optimum estimated mass is found to be rather tedious and time-consuming. It may be solved by using intelligent methods such as knowledge or rule based, iterative learning algorithm, fuzzy logic, neural network, or their hybrids. Other operating and loading conditions such as varied trolley and load masses, length of cable and speed of trolley traverse could also be experimented and tried to further explore the dynamic response of the system.

## **NOMENCLATURE**

$F_x$  is the force acting on the trolley  
 $F_1$  is the force acting on the hoisting cable  
 $m_1$  is the mass of trolley  
 $m_2$  is the mass of payload  
 $\ddot{l}$  is the hoisting cable extending acceleration  
 $\dot{l}$  is the hoisting cable extending speed  
 $l$  is the length of hoisting cable  
 $\ddot{x}$  is the acceleration of trolley  
 $\ddot{\theta}$  is the acceleration of swaying of payload

$\dot{\theta}$  is the speed of swaying of payload  
 $\theta$  is the sway angle of payload  
 $g$  is the gravity acceleration  
 $V(t)$  is the supplied voltage  
 $R$  is the electric resistance  
 $L$  is the electric inductance  
 $I(t)$  is the current in the electric circuit  
 $K_m$  is the motor constant  
 $T(t)$  is the torque produced  
 $K_t$  is the torque constant  
 $J$  is the inertia of the motor  
 $b$  is the damping coefficient associated with the mechanical system  
 $F_E$  is the force supplied  
 $r$  is the radius of gear used  
 $F$  is the acting force  
 $m$  is the mass of the trolley  
 $a$  is the acceleration of the trolley  
 $K_P$  is the proportional gain  
 $K_I$  is the integral gain  
 $K_D$  is the derivative gain  
 $F_d^*$  is the estimated disturbance force  
 $F_q$  is the applied force from the actuator (obtained through measurement)  
 $M$  is the estimated mass in the AFC loop obtained using appropriate methods  
 $\ddot{x}$  is the acceleration of the trolley (obtained through measurement)

## REFERENCES

1. <http://www.low-cost-cranes.com/images/GantryCrane4.jpg>, May 2008.
2. Rahman, E.M.A., Nayfeh, A.H., Masoud, Z.N., 2003. Dynamic and Control of Crane: a Review, *Journal of Vibration and Control*, 9(7), 863-908.
3. Daqaq M.F. and Masoud, Z.N., 2006. Nonlinear Input-Shaping Controller for Quay-Side Container Cranes, *Journal of Nonlinear Dynamics*, 45, 149–170.
4. Saigo, M., Tanaka, N., Tani, K., 1998. An Approach to Vibration Control of Multiple-Pendulum System by Wave Absorption, *ASME Journal of Vibration and Acoustic*, 120, 524–533.
5. Toxqui, R.T., Yu, W., Li, X., 2006. PD Control of Overhead Crane Systems with Neural Compensation, *Lecture Notes in Computer Science*, Springer Berlin/Heidelberg, 1110-1115.
6. Wahyudi and Jalani, J., 2007. Hardware-In-The-Loop Simulation (HILS) Based Design and Evaluation of an Intelligent Gantry Crane System, *International Journal of Automation Technology*, 1(1), 11-19.
7. Masoud, Z.N., Nayfeh, A.H., Nayfeh, N.A., 2005. Sway Reduction on Quay-side Container Cranes Using Delayed Feedback Controller: Simulations and Experiments, *Journal of Vibration and Control*, 11(8), 1103-1122.

8. Hewit, J. R. and Burdess, J.S., 1981. S. Fast, Dynamic Decoupled Control for Robotics using Active Force Control, *Trans. Mechanism and Machine Theory*, 16(5), 535-542.
9. Mailah, M., 1998. *Intelligent Active Force Control of A Rigid Robot Arm Using Neural Network and Iterative Learning Algorithms*, PhD Thesis, University of Dundee, UK.
10. Mailah, M., Pitowarno, E., Jamaluddin, H., 2005. Robust Motion Control for Mobile Manipulator Using Resolved Acceleration and Proportional-Integral Active Force Control, *International Journal of Advanced Robotic Systems*, 2(2), 125–134.
11. Kwek, L. C., Wong, E. K., Loo, C.K., Rao, M.V.C., 2003. Application of Active Force Control and Iterative Learning in a 5-Link Biped Robot. *Journal of Intelligent and Robotic Systems*. 37, 143-162.
12. Hudha, K., Jamaluddin, H., Samin, P.M., Rahman, R.A., 2003. Active Roll Control Suspension System Using Active Force Control Strategy on a New Modified Half Car Model, *Proceeding of Society of Automotive Engineers*, SAE- 03FTT-15.
13. Priyandoko, G., Mailah, M., Jamaluddin, H., 2008, Vehicle Active Suspension System Using Skyhook Adaptive Neuro Active Force Control, *Mechanical Systems and Signal Processing*, doi:10.1016/j.ymsp.2008.07.014.
14. Hewit, J.R. and Burdess, J.S., 1986. An Active Method for the Control of Mechanical Systems in the Presence of Unmeasurable Forcing, *Trans. on Mechanisms and Machines Theory*, 21(3), 393-400.