

SOLAR POWER PROFILE PREDICTION FOR LOW EARTH ORBIT SATELLITES

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ABSTRACT

This paper presents the development of algorithms using MATLAB® codes to predict the in-orbit satellite power profile. Satellite power requirement is a crucial parameter for its in-orbit operation. In this case, it is best to identify the power profile which indicates the amount of power generated over a time frame of an orbit. However, the determination of the satellite power profile requires substantial amount of efforts to compute and this is largely due to the complex numerical treatments. Orbital parameters are deemed to affect the determination of the satellite power profile. Therefore, a computer program has been written to solve all the governing equations leading to the satellite power profile prediction for an orbit and eventually for a year. The power profile validation was done analytically using the governing equations before the profile is generated through the computer codes. This work contributes greatly towards the small satellite (<100 kg) power sizing effort and eliminates the need of a costly commercial power sizing software.

Keywords: *Orbit, power profile, small satellite design.*

1.0 INTRODUCTION

Satellites are spacecraft that receive signals and send them back to Earth. Satellite's can be seen operating for global telecommunication industry which provides communication coverage on Earth [1].

To enable various missions, Low Earth Orbit (LEO) satellites are required to fulfill their requirements on power consumption to sustain their life in space. As typical ones use photovoltaic cells to generate solar power, the presence of Sun phase is important. Apart from the issue of having the Sun to generate power, a greater concern shall also be placed on its eclipse duration, when generation of solar power is impossible.

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Hence, the position of a satellite in an orbit dictates its power profile. In any time within its life, a solar powered satellite may fall into either the Sun or eclipse region [2]. Satellites position and their orientation in space are well represented using position vectors. A vector is represented in form of a matrix. Spatial computations using rotation vectors and matrices provide information regarding orientation of solar panels towards the Sun [3]. The orientation deduced as Sun angle β therefore dictates the instantaneous incident power P over an orbit period [4].

Corrections are imposed to the value P accordingly whenever the normal vector of solar panel surface faces against the sunlight ($\beta > 90^\circ$). Also, the power profile is not without some reduction due to inefficiencies. The efficiencies for various types of solar panels were evaluated according to materials and are given in several references [5, 6, 7, 8].

Power profile analysis may be difficult when it considers large duration of orbits or even orbits for a year. Hence, this paper will explain the development of a computer program which performs those analyses [9]. This should overcome the mathematical complexity and save considerable work time and effort.

2.0 MATHEMATICAL MODELS AND ALGORITHMS

This section describes various mathematical models and formulas, which are implemented in the power prediction algorithms. The introduction of rotation vectors, matrices and governing equations into the algorithms is shown towards the end of this section.

2.1 Governing Equations

A list of governing equations is given as follows:

$$i. \text{ Satellite angular velocity; } \omega_s = \left(\frac{\mu_e}{R_s^3} \right)^{\frac{1}{2}} \quad (1)$$

$$ii. \text{ Satellite orbit period; } T_o = \frac{2\pi}{\omega_s} \quad (2)$$

$$iii. \text{ Earth's angular change about the Sun per orbit; } \varphi = 2\pi \left(\frac{T_o}{T_y} \right) \quad (3)$$

$$iv. \text{ Satellite's angular change per minute; } \varphi_{sat} = 2\pi \left(\frac{T}{T_o} \right) \quad (4)$$

$$v. \text{ Sun angle; } \cos \beta_{i,m} = Np_i \bullet S_{sat,m} \quad (5)$$

$\beta_{i,m}$ a dot product of i-th and m-th elements of vectors Np with S_{sat} respectively. The index i is an integer which ranges from 1 to p . β ranges from zero to $\pi/2$.

vi. Instantaneous incident power; $P = \eta_y A_p \bullet S \bullet \cos \beta$ (6)

vii. Eclipse entry time; $T_{E,entry} = T_{Epeak} - \frac{\omega_s T_E}{2}$ (7)

viii. Eclipse exit time; $T_{E,exit} = T_{Epeak} + \frac{\omega_s T_E}{2}$ (8)

2.2 Rotation Vectors

A list of rotation vectors is given as follows:

i. Rotation vector of Earth about the Sun; $E_n = \begin{pmatrix} -\sin T_n \\ -\cos T_n \\ 0 \end{pmatrix}$ (9)

T_n , the n-th element of an array of Earth's rotation about the Sun and ranges between zero to 2π for every steps of ϕ .

ii. Rotation vector of satellite about the Earth; $B_m = \begin{pmatrix} \cos U_m \\ \sin U_m \\ 0 \end{pmatrix}$ (10)

U_m , the m-th element of an array of satellite's rotation about the Earth, ranges from zero to 2π for every steps of ϕ_{sat} .

iii. First rotation about Z axis;

$$A_\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\Omega) & \sin(\Omega) \\ 0 & -\sin(\Omega) & \cos(\Omega) \end{pmatrix} \quad (11)$$

iv. Second rotation about X axis; $A_n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i_n) & \sin(i_n) \\ 0 & -\sin(i_n) & \cos(i_n) \end{pmatrix}$ (12)

$$v. \text{ Third rotation about X axis; } A_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & \sin(i) \\ 0 & -\sin(i) & \cos(i) \end{pmatrix} \quad (13)$$

vi. Rotation of the Sun about the satellite in SCO frame;

$$S_{sat,m} = \begin{pmatrix} -\sin \theta_{s,m} & \cos \theta_{s,m} & 0 \\ 0 & 0 & -1 \\ \cos \theta_{s,m} & \sin \theta_{s,m} & 0 \end{pmatrix} \bullet S_e \quad (14)$$

$S_{sat,m}$, the m-th element of S_{sat} array. SCO frame axes are defined as follows:

- a. X-axis points along flight direction
- b. Y-axis is normal to flight direction
- c. Z-axis points towards center of the Earth (nadir)

2.3 Algorithms

The GUI allows four main options to commence satellite power profile analyses. Those options are to compute the satellite solar power generation; present the power profile per orbit, and per year; as well as the power requirement per year.

Upon activation of the COMPUTE pushbutton, the program will perform the following procedures in steps:

- Step 1: The program calls for Algorithm 1 (see Figure. 1) and display the information in MATLAB® Workspace.
- Step 2: The program calls for Algorithm 2 (see Figure. 2) and display the information in MATLAB® Workspace.
- Step 3: The program calls for Algorithm 3 (see Figure. 3) and display the information in MATLAB® Workspace.
- Step 4: The program imposes Earth's rotation about the Sun for 10° to 360° with increment of 10°. For each increment, the following steps are then followed:
 - i. The Sun Phase is set from 1 to its maximum count (obtained from Algorithm 3) with steps of 100 unit.
 - ii. For each step, the program calls for Algorithm 4 (see Figure. 4) and then Algorithm 5 (see Figure. 5). Information from the both algorithms is displayed in MATLAB® Workspace.
- Step 5: The program calls for Algorithm 6 and display the information in MATLAB® Workspace.
- Step 6: The program saves all information in MATLAB® Workspace into .MAT file.

Upon activation of the POWER PROFILE PER YEAR pushbutton, the program will perform the following procedures in steps:

- Step 1: The program attempts to load previous information.
- Step 2:
 - i. If information is not available, then **error warning** is displayed.

- ii. Else, the program produces a **surface plot for the power profile per year**. The figure is defined with the following particulars:
 - a. X-axis labeled “Duration of Orbit [min]”
 - b. Y-axis labeled “Rotation about the Sun [°]”
 - c. Z-axis labeled “Power [W]”
 - d. Title labeled “Generated Solar Power for a Year”

Upon activation of the *POWER REQUIREMENT PER YEAR* pushbutton, the program will perform the following procedures in steps:

- Step 1: Program will attempt to load previous information.
- Step 2:
 - i. If information is not available, then **error warning** is displayed.
 - ii. Else, the program will produce a **line plot for minimum power requirement per year**.
 - iii. The figure is defined with the following particulars:
 - a. X-axis labeled “Rotation about the Sun [°]”
 - b. Y-axis labeled “Power [W]”
 - c. Title labeled “Power Requirement for a Year”

Upon activation of the *POWER PROFILE PER ORBIT* pushbutton, the program will perform the following procedures:

- Step 1: Program will attempt to load previous information.
- Step 2:
 - i. If information is not available, then **error warning** is displayed.
 - ii. Else, the program will request user for position of the Earth rotation about the Sun. Then it will produce a **line plot for the power profile per orbit**. The figure will be defined with the following particulars:
 - a. X-axis labeled “Duration of Orbit [min]”
 - b. Y-axis labeled “Power [W]”
 - c. Title labeled “Satellite Power Profile Per Orbit”

All the algorithms for the in-orbit satellite power profile generation program are shown in Figures 1 to 6.

```

h = str2double(inputdlg('Orbit altitude', 'Input'))           - unit [km]
i_n = str2double(inputdlg('Orbit inclination', 'Input'))       - unit [°]

ηPV = str2double(inputdlg('PV efficiency', 'Input'))
ηT = str2double(inputdlg('Temperature effect', 'Input'))
ηdesign = str2double(inputdlg('Design & Assembly', 'Input'))
ηS = str2double(inputdlg('Shadowing factor', 'Input'))

ηy = ηPV • ηS • ηT • ηdesign                               - total solar panel efficiencies
Call Algorithm 7
    
```

Figure 1 : Algorithm 1 requests user for information regarding the satellite’s orbit parameters as well as additional data.

$R_s = R_E + (h \times 1000)$	– radius of orbit
Equation (1)	– satellite angular velocity
Equation (2)	– satellite's orbit period

Figure 2: Algorithm 2 computes orbit period of the satellite.

Equation (3)	– Earth's angular change about the Sun per orbit φ
The value φ is used to create an arithmetic sequence with elements T_n :	
$T_n = \varphi : \varphi : 2\pi$	– $T_n = T_1, T_2, T_3 \dots T_{max} = \varphi, 2\varphi, 3\varphi \dots 2\pi$
Equation (9)	– rotating vectors for the Earth about the Sun E_n

Figure 3: Algorithm 3 determines rotation of the Earth about the Sun.

$i = -23.5^\circ$	– inclination of the Earth about the ecliptic plane:
Equation (4)	– satellite's angular change about the Earth per minute φ_{sat}
The value φ_{sat} is used to create an arithmetic sequence with elements U_m :	
$U_m = \varphi_{sat} : \varphi_{sat} : 2\pi$	– $U_m = U_1, U_2, U_3 \dots U_{max} = \varphi_{sat}, 2\varphi_{sat}, 3\varphi_{sat} \dots 2\pi$
Each number in this sequence represents the shift in satellite's position as it rotate about the Earth for each one minute.	
Equation (10)	– rotating vector for the satellite about the Earth B_m
The reference plane for all rotating vectors is transformed to ecliptic plane by employing transformation matrices shown by Eqs. (11) to (13).	
$S_p = A_i \bullet A_n \bullet A_\Omega \bullet B_m$	– B_m with reference to the ecliptic plane
$S_e = A_i \bullet A_n \bullet A_\Omega \bullet -E_n$	– E_n with reference to the ecliptic plane
Equation (14)	– The rotating vector for the satellite about the Sun S_{sat} in the SCO frame

Figure 4: Algorithm 4 determines rotation of the Sun about the satellite with reference to ecliptic plane

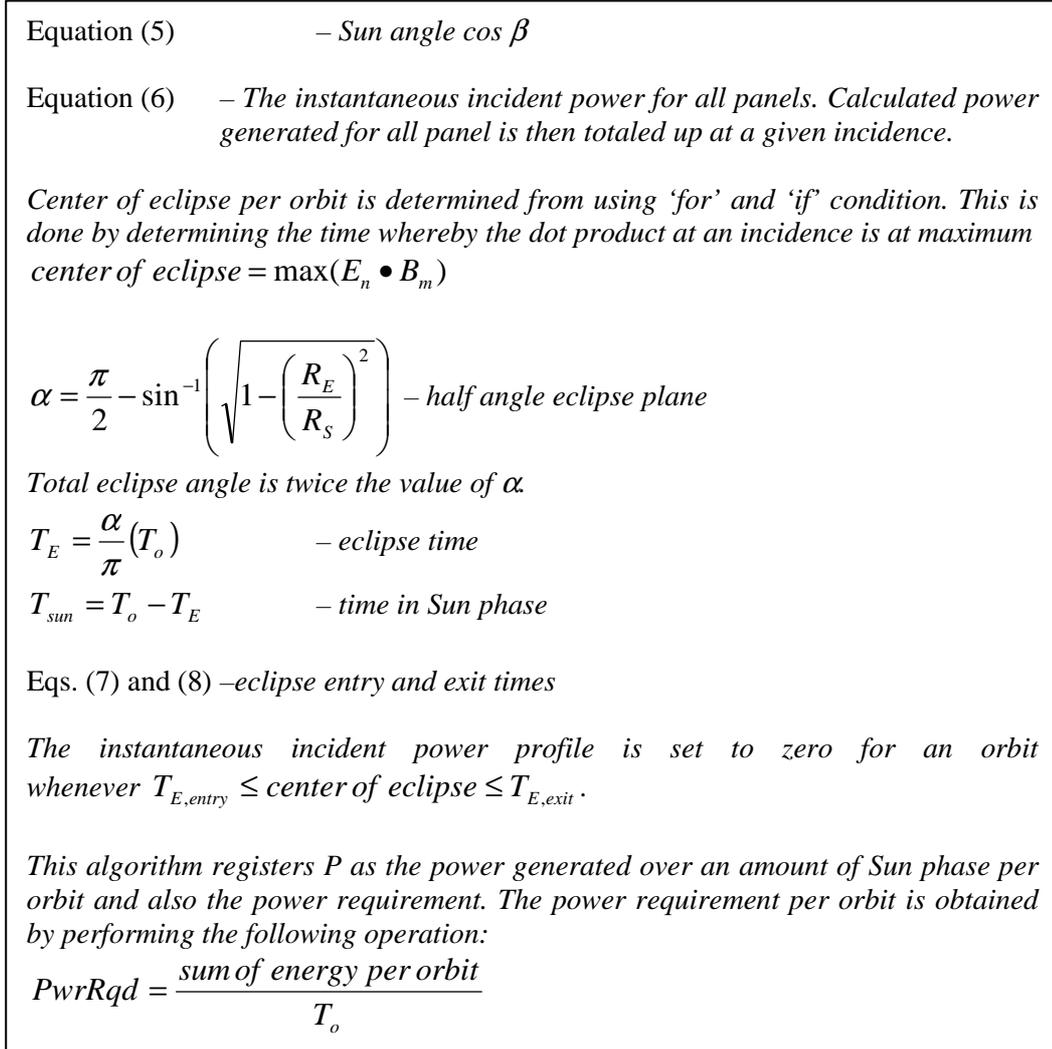


Figure 5: Algorithm 5 analyses the satellite’s power profile for each orbit.

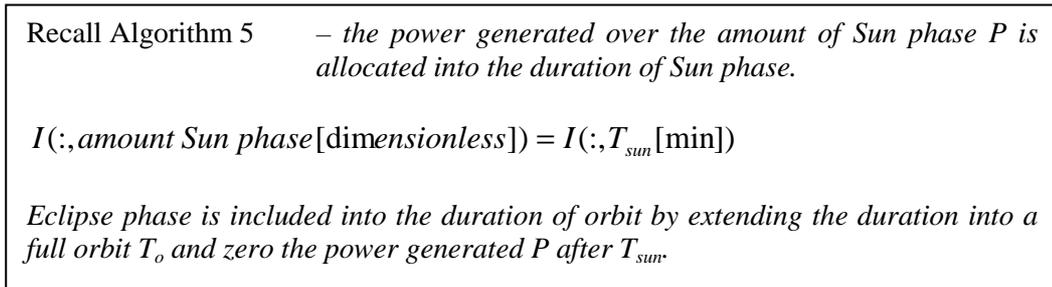


Figure 6: Algorithm 6 functions by computing power profile for a year.

The algorithms shown in Figures 1 to 6 are operated in sequence as shown in the block diagrams in Figure 7.

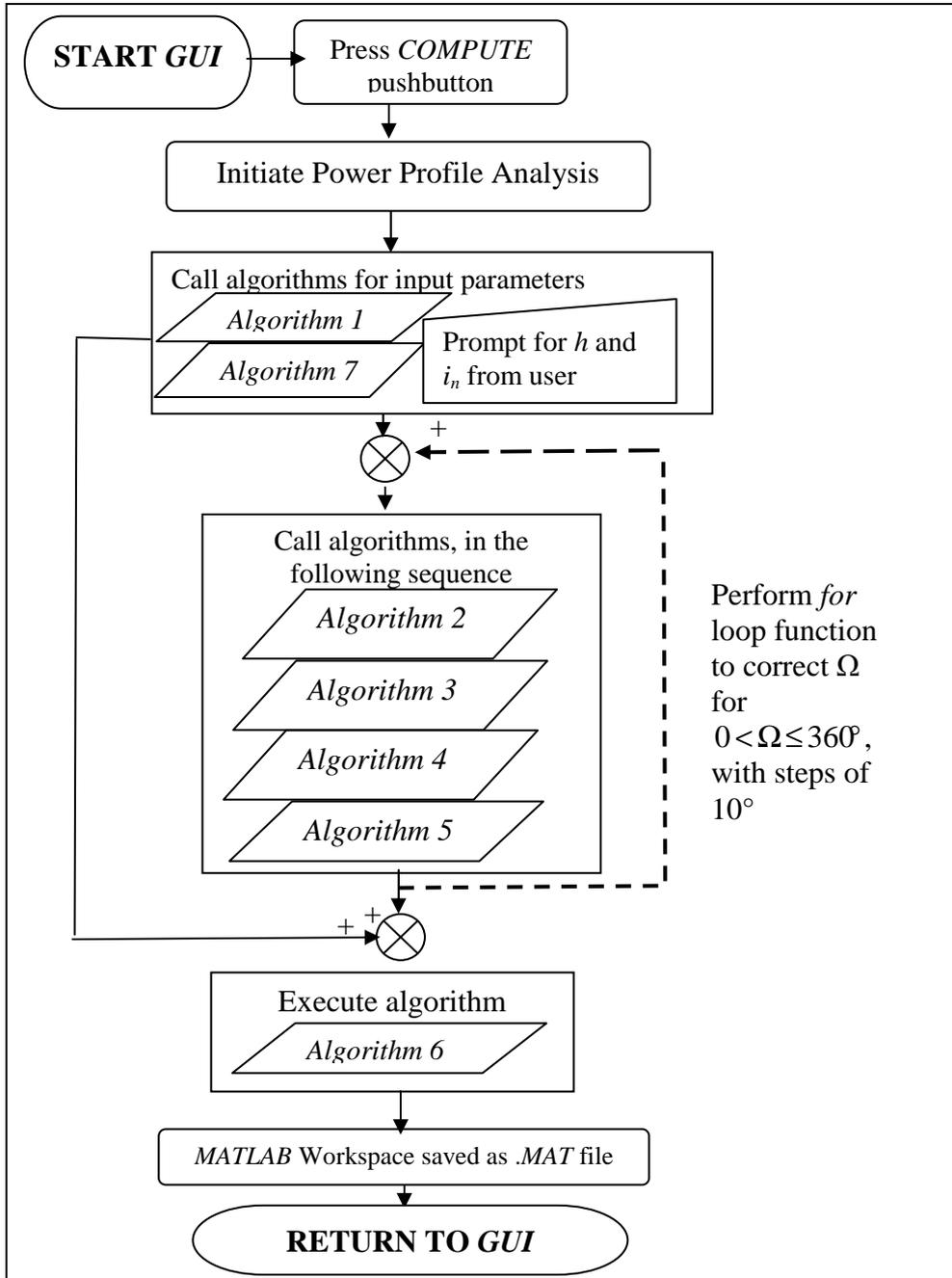


Figure 7 : Flow chart shows process path to compute generated power by pressing *COMPUTE* pushbutton

The flow chart in Figure 7 is a brief description of data flow and processes undertaken by the program to determine the satellite power profile for an orbit and a year. The display of the satellite power profiles is achieved by processes shown in Figure 8.

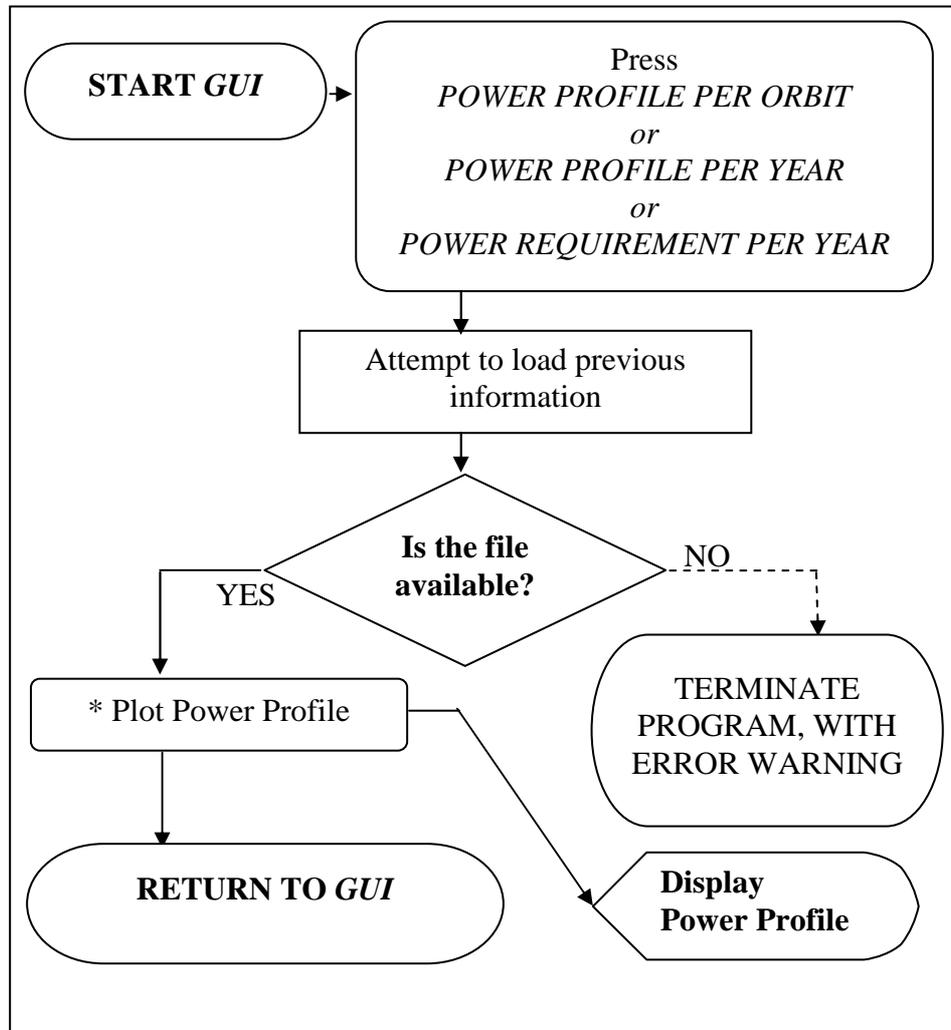


Figure 8 : Flow chart shows process path to display either power profile per orbit; power profile per year; or power requirement per year

To display the satellite power profile per-orbit, the program shall prompt the user for the Earth's position about the Sun. This additional process is embedded in a block marked with an asterisk (*) in Figure 8.

3.0 RESULTS AND DISCUSSION

This section presents and discusses the generated in-orbit satellite power profile per orbit and per year. The following power profiles were the outcome from using different input parameters as indicated along with the figures. Note that the results consider a cubic satellite body with solar panel arrangements as shown in Figure 9.

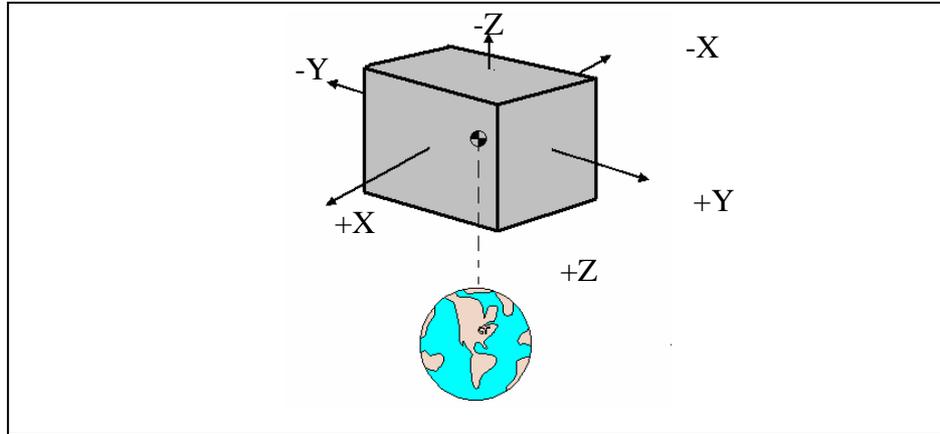


Figure 9 : The arrows indicate each with 0.048 m^2 area and a solar panel Efficiency factor of 0.15

The power profile validation was done analytically (Figure. 10 (a)) using the all governing equations before the power profile is generated through the computer codes (Figure. 10 (b)). The peak power in both cases is the same; thus, the computer codes are validated.

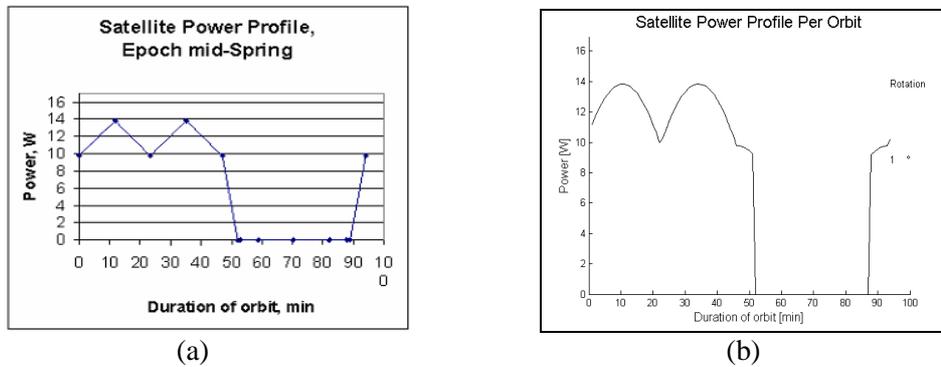


Figure 10: Satellite power profile per orbit at $h = 500 \text{ km}$; $i_n = 23.5^\circ$

3.1 Power Profile Per Year

Figures 11 (a), (b), (c) and (d) are generated as the result from the using different altitude and inclination parameters. Satellite power profile generation for a year provides information on the amount of solar power generated over the duration of orbit in a year. The solar power generated is quantified in Watts whereas other parameters such as the duration of orbit and steps in a year are given in minutes and degrees angle, respectively.

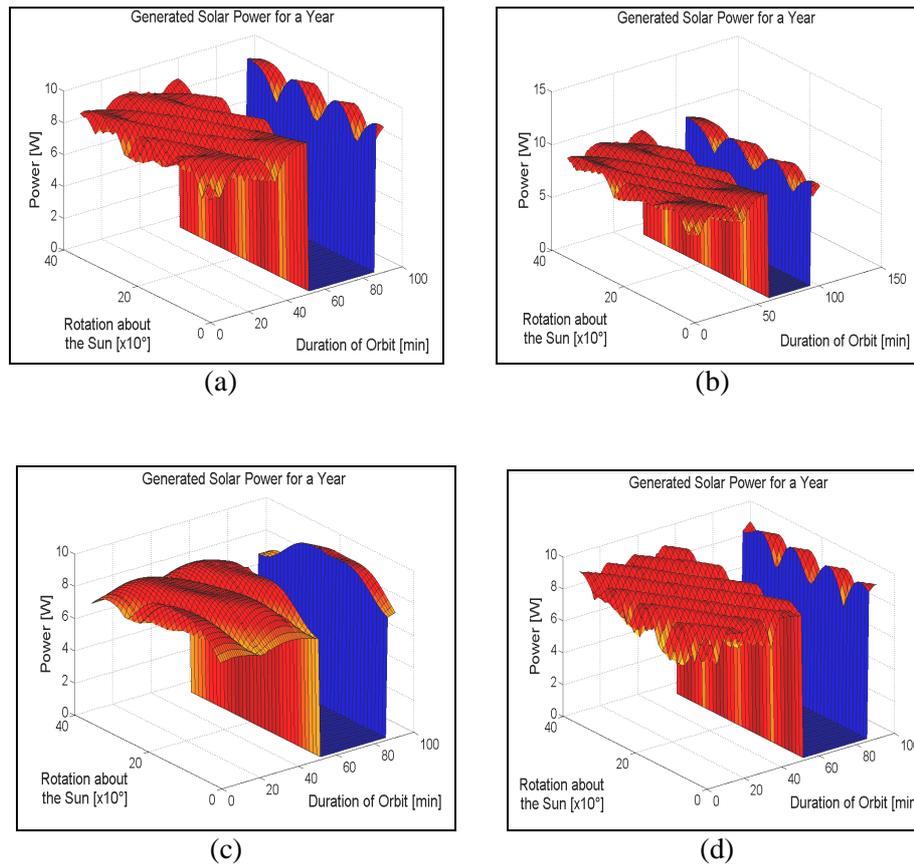
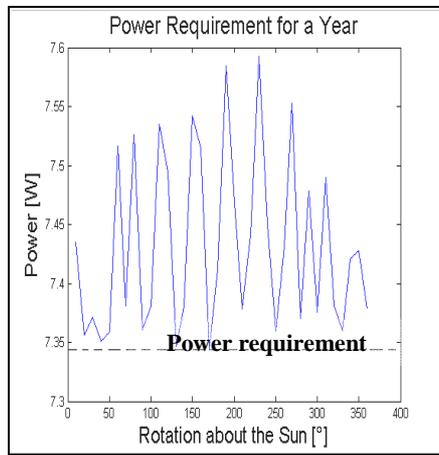


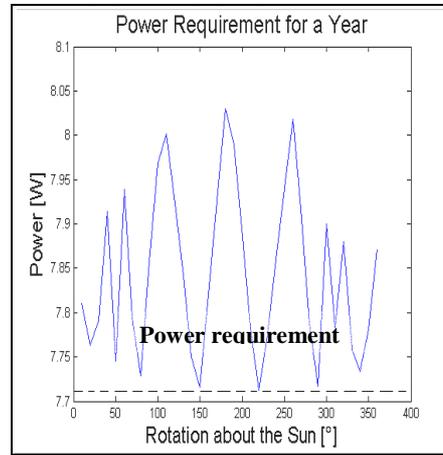
Figure 11 : 3D Satellite Power Profile Generation simulated for a year at; (a) $h = 500$ km with $i_n = 53^\circ$; (b) $h = 1,000$ km with $i_n = 53^\circ$; (c) $h = 500$ km with $i_n = 23.5^\circ$; and (d) $h = 500$ km with $i_n = 82.5^\circ$

3.2 Power Requirement Per Year

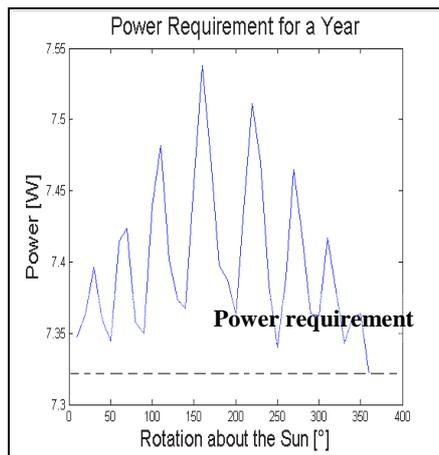
Having the 3D power profile per year, the power requirement for power sizing works can be determined as shown in Figures. 12 (a), (b), (c), and (d). Note that the satellite power sizing is done by considering the lowest value, as indicated by the dashed lines.



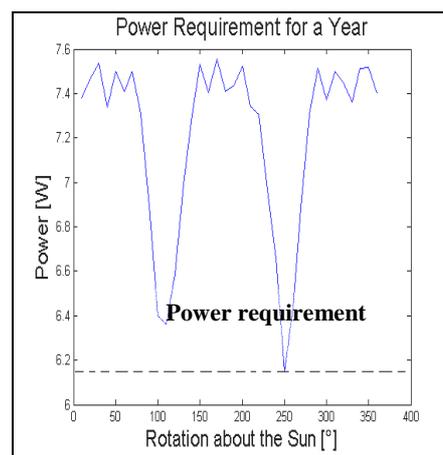
a) $h = 500$ km with $i_n = 53^\circ$;



b) $h = 1,000$ km with $i_n = 53^\circ$



c) $h = 500$ km with $i_n = 23.5^\circ$

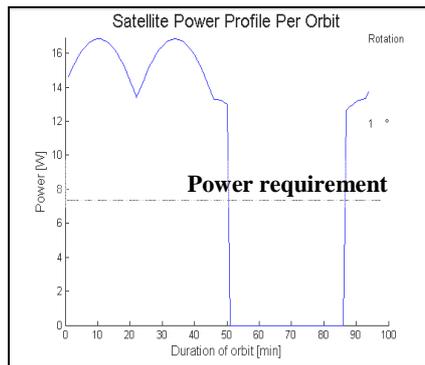


d) $h = 500$ km with $i_n = 82.5^\circ$

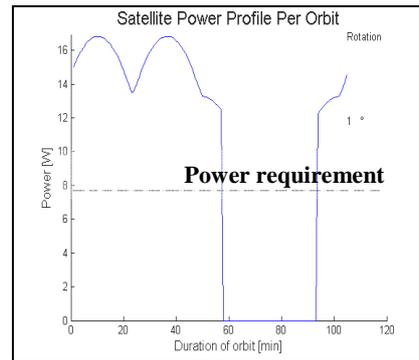
Figure 12: Satellite Power Profile Generation for in-orbit satellite simulated for power requirement in a year.

3.3 Power Profile Per Orbit

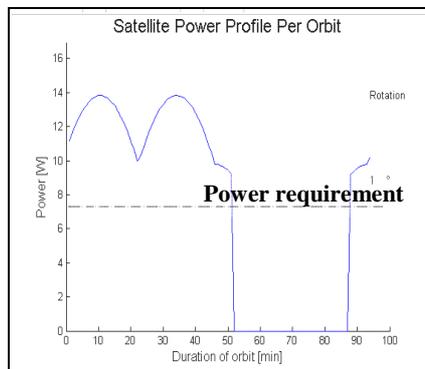
After setting the power requirement, the satellite power profile for an orbit can be viewed as well. Figures. 13 (a), (b), (c) and (d) show the corresponding power profile per orbit. Note that the satellite power requirement limit is indicated by the dashed lines as shown in Figures 13 (a), (b), (c), and (d).



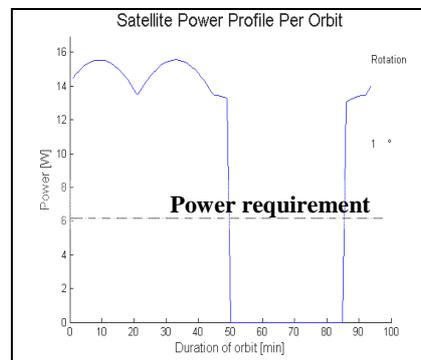
a) $h = 500$ km with $i_n = 53^\circ$



b) $h = 1,000$ km with $i_n = 53^\circ$



c) $h = 500$ km with $i_n = 23.5$



d) $h = 500$ km with $i_n = 82.5^\circ$

Figure 13: 2D Satellite Power Profile Generation for in-orbit satellite simulated for per orbit.

3.4 Discussion

The numerical and simulation results obtained from this program are useful in the mission analysis and the small satellite power sizing. Based on the orbit inclination, altitude and solar panel sizes, the computer program can generate the in-orbit power profile and the mission power requirement.

By using this program, the time consuming analytical method can be replaced by this numerical method for the solar power profile generation. A user friendly *GUI* program enables the input parameters such as altitude, orbit inclination and photovoltaic cell efficiencies for the program. The developed program is validated analytically using the power profiles generated per-orbit.

4.0 CONCLUSION

A computer code was developed using MATLAB® to simulate the in-orbit small satellite power profile. By defining the mission parameters such as the altitude, orbit inclination and photovoltaic cell efficiencies, the program generates the in-orbit solar power profiles. Initially, the in-orbit simulation for an orbit was successfully implemented. The simulated result was validated using the result obtained analytically. Apparently, both results were found to be similar. Eventually, the generation of power profile for an orbit was successfully extended to simulate an in-orbit 3D satellite power profile for one complete year orbit. As a conclusion, the developed program successfully determines the in-orbit power profiles as well as the power requirement for different small satellites (< 100 kg).

NOMENCLATURES

A_p	Photovoltaic cell surface area [m ²]
B	Satellite's position vectors in orbit plane [m]
E	Earth's position vector centered at the Sun [m]
ESA	European Space Agency
GUI	Graphical User Interface
GUIDE	Graphical User Interface Development Environment
h	Altitude [m]
i	Earth inclination [deg]
i_n	Satellite orbit inclination [deg]
φ	Angular step change [rad]
LEO	Low Earth Orbit
N_p	Normal vector of solar panel
P	Power [W]
p	Number of solar panel
R_S	Radius of orbit from center of the Earth [m]
R_E	Radius of the Earth, by default is 6,378.140 km
S	Solar constant, by default is 1,358 W/m ²
SCO	Spacecraft Centered Orbit
S_e	Satellite position vector (Earth centered) [m]
T	Step change in time [1 min = 60 s]
T_E	Duration of eclipse [s]
T_o	Orbit Period of a satellite about the Earth [s]
T_{sun}	Duration of Sun phase [s]
T_y	Orbit Period of the Earth about the Sun, by default is 31,588,149.548 s
η_{design}	Structural and design efficiency [%]
η_{pv}	Photovoltaic cells efficiency [%]
η_S	Shading factor [%]
η_T	Thermal efficiency [%]
η_y	Solar panel efficiency [%]
Ω	Right ascension of ascending node [deg]

ω_s	Angular velocity of satellite about Earth [rad/s]
μ_e	Earth gravitational constant, is $3.986005 \times 10^{14} \text{ m}^3/\text{s}^2$
α	Half-angle of eclipse region [deg]
β	Angle between the Sun vector and surface normal vector [deg]

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